Non-Equidistant Grey Model Based on Background Value and Initial Condition Optimization and Its Application

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ABSTRACT

As the background value and the initial condition are important factors affecting the precision of grey system model, in this paper, we put forward an improved non-equidistant GM(1,1) model based on PSO according to the practical application need, in which the background value is optimized firstly and a new initial condition is presented based on the principle of new information priority. Under the algorithm of minimizing the square sum of the relative error between the original series and the forecasting sequences, the solution to the optimized time parameter is given, the particle swarm optimization (PSO) algorithm is used as a tool to optimize the parameter in the background value and the initial condition. The experimental result shows the effectiveness and applicability of the proposed non-equidistant GM(1,1) model.

CCS CONCEPTS

• **Computing methodologies**; • **Modeling and simulation**; • **Model development and analysis**;

KEYWORDS

Non-equidistant GM(1,1) model, Background value, Initial condition, Particle swarm optimization

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1 INTRODUCTION

Grey prediction is an important part of grey system theory. Due to process and small samples, it has been successfully applied to electric power, agriculture, management, economy, high-tech industry and various fields [1-5]. There are many kinds of grey prediction models, among which GM(1,1) model is the most widely used and the most important grey prediction model. It is a prerequisite for GM(1,1) model that the original sequence meets the condition of equal time interval. However, due to many reasons, in practice, there are many incomplete original data and non-equal time interval sample sequences, such as material fatigue measure data[6], dam settlement data[7], and so on. So, it is necessary to establish non-equidistant GM(1,1) model. In order to improve the performance of the traditional non-equidistant GM(1,1) model, some optimization researches have been carried out. Aiming at the nonlinear non-equidistant problem existed, Wang[8] proposed a non-equidistant GM(1,1) power model, and achieved a good effect in the fatigue strength prediction of titanium alloy; Zheng[9] presented a parameter estimation optimization method of non-equidistant GM(1,1) model, and predicted the demand for urban construction land accurately; Guo[10] proposed a non-equidistant GM(1,1,t$^{\alpha}$) model with time power term, in which the relationship between development coefficient and power exponent is studied, particle swarm optimization algorithm (PSO) is applied to optimize the power exponent; Shen[11] proposed a fractional-order non-equidistant GM(1,1) model, and applied Levenberg-Marquardt algorithm for the parameter optimization to enhance the prediction ability of the model;Luo[12] proposed a non-equidistant GM(1,1) model with$x^{(1)}(k_n)$ as the initial condition based on the principle of new information priority. Xiong[13] took the weighted average of each component of the first-order accumulative generation operator (1-AGO) sequence as the initial condition and constructed an optimized non-equidistant GM(1,1) model; Xi[14] designed a new initial condition by using the weighted sum of each component in 1-AGO sequence based on the principle of the new information priority. In this paper, based on the existing research, a non-equidistant GM(1,1) optimization model based on PSO is proposed, the background value is reconstructed by the compound trapezoid formula; based on the principle of the new information priority, an variable weighted initial condition with time parameter is proposed. Under the algorithm of minimizing the square sum of the relative error between the original sequences and the forecasting sequences, the solution to the optimized time parameter is given, the particle swarm optimization (PSO) algorithm is used as a tool to optimize the parameters in the background value and the initial condition. The experimental result shows the effectiveness and applicability of the proposed non-equidistant GM(1,1) model.

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2 THE CONSTRUCTION OF NON-EQUIDISTANT GREY MODEL BASED ON PSO

2.1 The Traditional Non-Equidistant GM(1,1) Grey Model

Theorem 1. Assuming the original data sequences is: $X^{(0)} = (x^{(0)}(k_1), x^{(0)}(k_2), \cdot s, x^{(0)}(k_n))$, the 1-AGO sequences is:

$$X^{(1)} = (x^{(1)}(k_1), x^{(1)}(k_2), \cdot s, x^{(1)}(k_n))$$

$$x^{(1)}(k_i) = \sum_{j=1}^{i} x^{(0)}(k_j) \quad \Delta k_j \quad , i = 1, 2 \cdot sn$$

$$\Delta k_j = k_j - k_{j-1} \neq const, \Delta k_1 = 1$$

The background value $Z^{(1)}$:

$$Z^{(1)} = (z^{(1)}(k_2), z^{(1)}(k_3), ..., z^{(1)}(k_n))$$

$$z^{(1)}(k_i) = 0.5\left(x^{(1)}(k_i) + x^{(1)}(k_{i-1})\right),$$

$$i = 2, 3, ..., n$$

The whitening equation of the traditional non-equidistant GM(1, 1) model is defined as:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b$$

The difference equation is defined as:

$$x^{(0)}(k_i)\Delta k_i + az^{(1)}(k_i) = b$$

where a is the developing coefficient, b is called the driving coefficient.

If

$$Y = \begin{pmatrix} x^{(0)}(k_2) \\ x^{(0)}(k_3) \\ \vdots \\ x^{(0)}(k_n) \end{pmatrix} \quad B = \begin{bmatrix} -z^{(1)}(k_2) & 1 \\ -z^{(1)}(k_3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(k_n) & 1 \end{bmatrix},$$

then we can obtain: $\hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y$.

Theorem 2. The time response function $\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b$ is:

$$x^{(1)}(t) = Ce^{-at} + b/a \tag{1}$$

If the initial condition is:$x^{(1)}(t)|_{t=k_1} = x^{(1)}(k_1)$, then, we can get the simulating and forecasting value:

$$\hat{x}^{(1)}(k_i) = \left(x^{(1)}(k_1) - b/a\right) e^{-a(k_i - k_1)} + b/a \tag{2}$$

$$\hat{x}^{(0)}\frac{(k_i) = \left(x^{(1)}(k_1) - b/a\right)\left(1 - e^{a\Delta k_i}\right) e^{-a(k_i - k_1)}}{\Delta k_i} \tag{3}$$

As can be seen from the above modeling process of the non-equidistant GM(1,1) model, the forecasting accuracy of the non-equidistant GM(1,1) model depends on the parameters: $\hat{a} = [a, b]^T$ and the initial condition, the value of the parameters depends on the solution of the background value $z^{(1)}(k_i)$, so the background value and initial condition are the two most important factors that have important effects on the modeling accuracy of the non-equidistant GM(1,1) model. Therefore, in this paper, we improve the performance of the traditional non-equidistant GM(1,1) model from the perspective of background value and initial condition, respectively.

2.2 Improved Background Value for Non-Equidistant GM(1,1) Model

In the traditional non-equidistant GM(1,1) model, the continuous function of the sequence $x^{(1)}(k_i)$ is considered as a linear function, so the area enclosed by $x^{(1)}(k_i)$and t-axis on the interval $[k_{i-1}, k_i]$ is replaced by the trapezoidal formula $0.5(x^{(1)}(k_i) + x^{(1)}(k_{i-1}))$, which obviously leads to errors. Because the solution of$x^{(0)}(k_i)\Delta k_i + az^{(1)}(k_i) = b$ is in a nonhomogeneous exponential form, if we assume$x^{(1)}(k_i)$can be expressed as a general inhomogeneous sequence [15]:

$$x^{(1)}(k_i) = Ne^{r(k_i - k_1)} + M$$

where:

$$r = \frac{\ln x^{(0)}(k_i) - \ln x^{(0)}(k_{i-1})}{\Delta k_i}$$

$$N = \frac{\Delta k_i x^{(0)}(k_i)}{(1 - e^{a\Delta k_i})e^{-a(k_i - k_1)}}$$

$$M = x^{(0)}(k_1) - N$$

In this paper, in order to obtain more accurate background value, we applied a compound trapezoid formula[15] to improve the background value of non-equidistant GM(1,1) model by the idea of function approximation:

$$\begin{aligned} z^{(1)}(k_i) &= \int_{k_{i-1}}^{k_i} x^{(1)} dt \\ &= \frac{1}{2p}[x^{(1)}(k_{i-1}) + x^{(1)}(k_i) + 2\sum_{i=1}^{p-1} x^{(1)}(k_{i-1} + iq)] \end{aligned}$$

where $q = \frac{k_i - k_{i-1}}{p}$, p is an adaptive parameter, p is an integer, $p \geq 1$.

2.3 The Existing Initial Condition Optimization Methods of Non-Equidistant GM(1,1) Model and Corresponding Defect Analysis

The existing initial condition optimization methods of the non-equidistant GM(1,1) model mainly fall into the following categories[14]:

(1) In literature [12], $x^{(1)}(t)|_{t=k_n} = x^{(1)}(k_n)$ is taken as the initial condition.

(2) In literature [13], $x^{(1)}(t)|_{t=\varphi} = \alpha_1 x^{(1)}(k_1) + \alpha_2 x^{(1)}(k_2) + \cdot s + \alpha_n x^{(1)}(k_n)$ is taken as the initial condition, $\alpha_1 + \alpha_2 + \cdot s + \alpha_n = 1$, $\alpha_i = k_i / \sum_{i=1}^{n} k_i$, φ is the time parameter.

(3) In literature [14], $x^{(1)}(t)|_{t=\rho} = \rho_1 x^{(1)}(k_1) + \rho_2 x^{(1)}(k_2) + \cdot s + \rho_n x^{(1)}(k_n) = \sum_{i=1}^{n} \rho_i x^{(1)}(k_i)$ is taken as the initial condition, $\rho_1 + \rho_2 + \cdot s + \rho_n = 1$, $\rho_i = x^{(1)}(k_i)^2 / \sum_{j=1}^{n} x^{(1)}(k_j)^2$, ρ is the time parameter.

In the above initial condition optimization methods, the vertical coordinate value of the optimized initial condition is deemed to be a certain value, such as: $x^{(1)}(k_n)$, $\alpha_1 x^{(1)}(k_1) + \alpha_2 x^{(1)}(k_2) + \cdot s + \alpha_n x^{(1)}(k_n)$, $\sum_{i=1}^{n} \rho_i x^{(1)}(k_i)$, however, whether these certain values are the optimal value of the initial value is lack of reasonable evidence, so a new initial condition optimization method is proposed in this paper.

Table 1: Accumulated Settlement Observation Values of A Laboratory Building

index	1	2	3	4	5	6	7	8	9	10
time(k_i)	1	25	53	83	116	147	177	237	269	355
observed value ($x^{(0)}(k_i)/mm$)	9.28	10.71	11.31	11.64	12	12.23	13.05	13.16	13.61	13.94

2.4 A Variable Weighted Initial Condition Based on Time Parameter

In view of the shortcoming of the above initial condition optimization methods, based on the principle of the new information priority, in this paper, we propose a vertical coordinate value optimization method with variable weighted initial condition based on time parameter, and the solution to the optimized time parameter is given under the algorithm of minimizing the square sum of the relative error between the original series and the forecasting sequences. The specific construction process is as follows:

$x^{(1)}(t)|_{t=\theta} = \delta x^{(1)}(k_n)$ whereθ is a time parameter, $\delta$is an adaptive parameter, $\frac{x^{(1)}(k_1)}{x^{(1)}(k_n)} \leq \delta \leq 1$.

Theorem 3. The optimal time parameter θ is:

$$\theta=\frac{1}{a}(\ln\sum_{i=1}^{n} Ge^{-ak_i} - \ln\sum_{i=1}^{n} G^2 e^{-2ak_i})$$

$$\mathrm{G} =[\delta x^{(1)}(k_n) - \frac{b}{a}](1 - e^{a\Delta k_i})\Big/[x^{(0)}(k_i)\Delta k_i]$$

Proof. If$X^{(0)} = (x^{(0)}(k_1), x^{(0)}(k_2), \cdot s, x^{(0)}(k_n))$is the original data sequences, $\hat{X}^{(0)} = (\hat{x}^{(0)}(k_1), \hat{x}^{(0)}(k_2), \cdot s, \hat{x}^{(0)}(k_n))$ is the simulating sequences, we can construct the square sum function of the relative error:

$$f(\theta) = \sum_{i=1}^{n}\left[\frac{\hat{x}^{(0)}(k_i)-x^{(0)}(k_i)}{x^{(0)}(k_i)}\right]^2 = \sum_{i=1}^{n}\left\{\frac{\left[\delta x^{(1)}(k_n)-\frac{b}{a}\right]\left(1-e^{a\Delta k_i}\right)e^{-a(k_i-\theta)}}{x^{(0)}(k_i)\Delta k_i} - 1\right\}$$

By minimizing the square sum of the relative error between the original series and the forecasting sequences, let $f(\theta)'$=0, we can get the optimal value of the time parameter:

$$\theta=\frac{1}{a}(\ln\sum_{i=1}^{n} Ge^{-ak_i} - \ln\sum_{i=1}^{n} G^2 e^{-2ak_i})$$

$$\mathrm{G} =[\delta x^{(1)}(k_n) - \frac{b}{a}](1 - e^{a\Delta k_i})\Big/[x^{(0)}(k_i)\Delta k_i]$$

Theorem 4. Under the initial condition: $x^{(1)}(t)|_{t=\theta} = \delta x^{(1)}(k_n)$, the simulating and forecasting value of $\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b$ is:

$$\hat{x}^{(1)}(k_i) = \left(\delta x^{(1)}(k_n) - b/a\right)e^{-a(k_i-\theta)} + b/a$$

$$\hat{x}^{(0)}(k_i) = \frac{\left(\delta x^{(1)}(k_n) - b/a\right)\left(1 - e^{\alpha\Delta k_i}\right)e^{-a(k_i-\theta)}}{\Delta k_i}$$

2.5 The Optimization of the Adaptive Parameters Based on PSO

In the algorithm presented in this paper, the adaptive parameters: p and δ are two key parameters which have significant impacts on the accuracy of simulation and forecasting. In this paper, PSO algorithm is applied to optimize the adaptive parameters based on the square sum of relative error. The square sum of relative error is shown as follows:

$$SSRE = \sum_{i=1}^{n}\left[\frac{\hat{x}^{(0)}(k_i) - x^{(0)}(k_i)}{x^{(0)}(k_i)}\right]^2$$

3 EXPERIMENTS AND RESULTS

In order to verify the validity and applicability of the proposed model in this paper, the settlement data of a teaching experimental building in literature [14] is selected, the data are shown in Table 1, it is obvious that the data have the non-equidistant characteristic. In this paper, the first 8 data are used as modeling, and the last 2 data are used for forecasting. We compare the performance of the proposed model with three frequently-used non-equidistant GM(1,1) models: the models in literature [12], [13] and [14]. The experimental results are shown in Table 2

Based on PSO, the optimal parameters of the propose model are: p=2, δ=0.7539. From Table 2, we can see that the one-step prediction relative error of the proposed model is 0.1207%, which is smaller than 3.1318% of the model in literature [12], 1.4987% of the model in literature [13], 0.6593% of the model in literature [14];the two-step prediction relative error of the proposed model is 2.7282%, which is smaller than 7.0929% of the model in literature [12], 5.3971% of the model in literature [13], 4.5409% of the model in literature [14];The mean forecasting relative error of the proposed model is 1.4245%, which is smaller than 5.2095% of the model in literature [12], 3.4479% of the model in literature [13], 2.6001% of the model in literature [14]. Therefore, compared with the non-equidistant model in literature [12], [13] and [14], the non-equidistant GM(1,1) model proposed in this paper based on PSO has better prediction accuracy.

4 CONCLUSION

As the background value and the initial condition are important factors affecting the precision of non-equidistant GM(1,1) grey system model, in this paper, according to the practical application need, an improved non-equidistant GM(1,1) model based on PSO is presented. The background value is reconstructed by the compound trapezoid formula. Based on the principle of new information priority, a new initial condition is presented, a solution of the optimal time parameter corresponding to the initial condition was given by

Table 2: Comparison of Simulated and Predicted Values by Different Methods

k_i	Actual value	Model in literature [12]		Model in literature [13]		Model in literature [14]		Model proposed in this paper	
		Simulation	relative error (%)	Simulation	relative error (%)	Simulation	relative error (%)	Simulation	relative error (%)
25	10.71	10.94	2.1063	10.76	0.4894	10.67	0.4006	10.69	0.1757
53	11.3	11.24	0.6594	11.06	2.2325	10.96	3.0922	10.97	2.9536
83	11.64	11.58	0.5197	11.40	2.0950	11.30	2.9490	11.30	2.8996
116	12	11.97	0.2897	11.78	1.8686	11.67	2.7170	11.67	2.7649
147	12.23	12.37	1.1452	12.17	0.4564	12.07	1.3092	12.05	1.4579
177	13.05	12.77	2.1556	12.57	3.7049	12.46	4.5227	12.42	4.7588
237	13.16	13.38	1.6877	13.17	0.0775	13.06	0.763	13.01	1.1488
269	13.61	14.04	3.1318	13.81	1.4987	13.70	0.6593	13.63	0.1207
355	13.94	14.93	7.0929	14.69	5.3971	14.57	4.5409	14.32	2.7282
mean forecasting relative error (%)		5.2095		3.4479		2.6001		1.4245	

minimizing the square sum of the relative error between the original series and the forecasting sequences, the adaptive parameters in the model were determined adaptively by PSO algorithm. The validity and practicability of the proposed model were verified by a numerical example.

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REFERENCES

[1] Deng JL (1982). Control problems of grey systems [J]. Systems and Control Letters. 1 (5): 288-294.
[2] Deng JL (2002). Foundation of grey theory [M]. Wuhan: Huazhong University of Science and Technology Press, 23-25.
[3] Liu SF, Dang YG, Fang ZG (2004). Grey system theory and its application [M]. Beijing: Science Press, 1-8.
[4] Ding S, Keith H, Dang Y G (2018). Forecasting China's electricity consumption using a new grey prediction model [J]. Energy, 149: 314-328.
[5] Ding S, Dang YG, Xu N, Wei L, Ye J (2017). Multi-variable time delayed discrete grey model [J]. Control and Decision, 32(11): 1997-2004.
[6] Li D C, Chang C J, Chen W C, *et al* (2011). An extended grey forecasting model for omnidirectional forecasting considering data gap difference[J]. Applied Mathematical Modelling, 35(10): 5051-5058.
[7] Wu BB, Chen L, Ge C (2012). Application of improved non-equidistance GM(1,1) model in dam settlement analysis [J]. Water Resources and Power. 30 (6): 95-97.
[8] Wang ZX, Dang YG, Liu SF (2012). Non-equidistant GM(1,1) power model and its application in engineering [J]. Strategic Study of CAE, 14(7): 98-102.
[9] Zheng B H, Luo L (2015). Non-equidistance optimum grey model GM (1,1) of requirement analysis of urban construction land and its empirical study[J]. Electronic Journal of Geotechnical Engineering, 20 (22): 1227-1232.
[10] Guo H, Xiao XP, Jeffrey F (2015). Non-equidistance GM(1,1,t^{α}) model with time power and its application [J]. Control and Decision, 30(8): 1514-1518.
[11] Shen Y, He B, Qin P (2016). Fractional-order grey prediction method for non-equidistant sequences[J]. Entropy, 18(6): 227-243.
[12] Luo YX (2010). Non-equidistant step by step optimum new information GM(1,1) and its application [J]. Systems Engineering-Theory & Practice, 30(12): 2254-2258.
[13] Xiong PP, Dang YG, Yao TX (2015). Modeling method of non-equidistant GM(1,1) model based on optimization initial condition [J]. Control and Decision, 30(11): 2097-2102.
[14] Xi L, Ding S, Xu N, Xiong PP (2019). Research on optimization of non-equidistant GM(1,1) model based on the principle of new information priority [J]. Control and Decision, 34(10): 2221-2228.
[15] Jiang SQ, Liu SF, Zhou XC (2014). Optimization of background value in GM(1,1) based on compound trapezoid formula [J]. Control and Decision, 29(12): 2221-2225.